

AdS₃ Solutions of IIB Supergravity from D3-branes

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ABSTRACT: We consider pure D3-brane configurations of IIB string theory which lead to supersymmetric solutions containing an AdS₃ factor. They can provide new examples of AdS₃/CFT₂ examples on D3-branes whose worldvolume is partially compactified. When the internal 7 dimensional space is non-compact, they can be identified as supersymmetric fluctuations of higher dimensional AdS solutions and are in general dual to 1/8-BPS operators thereof. We find that supersymmetry requires the 7 dimensional space take the form of a $U(1)$ fibration over a 6 dimensional Kahler manifold.

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1. Introduction

Due to the celebrated duality relation between string theory in anti-de-Sitter (AdS) backgrounds and conformal field theories on the boundary[1], it is nowadays of great interest to find new AdS solutions to string/M-theory and study their field theory duals. Several different avenues have been taken to accomplish this goal. For instance, when one replaces the spheres of the maximally supersymmetric examples by squashed spheres new AdS/CFT duality pairs can be obtained [2]. When the squashed sphere is a toric Sasaki-Einstein manifold, one can identify the dual gauge theory and nontrivial predictions of the duality relation can be shown to match the field theory results [3]. In particular, the field theory computation of Weyl anomaly using the a-maximization method [4] is shown to agree with the volume of toric Sasaki-Einstein manifolds through Z-minimization developed in [5]. Another approach which proved to be very fruitful is to use the lower-dimensional gauged supergravity theories as the springboard, i.e. first to find nontrivial AdS vacua in low dimensions and then uplift the solution to 10 or 11 dimensions. They correspond to fixed points of the renormalization group equation confronted when the maximally supersymmetric system is perturbed by a relevant operator. See, for instance Ref. [6].

When we restrict ourselves to supersymmetric solutions we can take a more systematic approach to study the generic forms of such vacua as in [7, 8]. Instead of trying to solve the Killing spinor equation employing a specific ansatz, one can derive the general form of the solution by making use of the information given by the Killing spinor equations. One constructs various spinor bilinears from the Killing spinor and the algebraic and differential conditions derived from the Killing spinor equation help to restrict the local form of the metric. For lower dimensional systems with less supersymmetry it is sometimes possible to classify all supersymmetric solutions. Several important class of new solutions have been found using this method. Supersymmetric black ring solutions and Sasaki-Einstein manifolds $Y^{p,q}$ [9, 10] are among them. A valuable insight into the gauge/gravity duality

has been obtained from the analysis of IIB string solutions containing two factors of three-spheres with unbroken supersymmetry [11]. They are dual to the 1/2-BPS operators of $N = 4$ super Yang-Mills theory and it was shown that a two-dimensional slice of the supergravity solution can be identified as fluctuations of free fermion phase space.

In this work we study IIB vacua with a AdS_3 factor and establish how supersymmetry restricts the local form of the metric and field fluxes. In M-theory, a comprehensive analysis for supersymmetric AdS_3 solutions has been undertaken in [12]. To simplify the analysis we consider pure D3-brane configurations, i.e. the nontrivial fields of IIB supergravity are only Ramond-Ramond 5-forms and metric. We find that the transverse 7-dimensional space takes the form of a $U(1)$ fibration over a complex 3-dimensional Kahler space. If the whole 7-dimensional space is compact we obtain a new AdS_3 vacua. Otherwise the solution should describe a BPS operator of higher dimensional conformal field theory, most likely 4-dimensional. In the next section we present our setup in detail, and present how the supersymmetry provides relations between various differential forms in the internal space. We also illustrate how the well-known AdS solutions can be reproduced when we choose appropriate 6 dimensional Kahler manifolds. We conclude with comments and suggestions for future works.

2. Ansatz

It is the goal of this work to study supersymmetric IIB solutions with a AdS_3 factor from D3-branes. Among the various fields of IIB supergravity we allow to turn on the metric and Ramond-Ramond five-forms only. We plan to derive how the existence of a nontrivial Killing spinor solution restricts the local form of the metric and the five-form field strength. Having fixed a 3 dimensional part of the metric, we get an effective system which is 7 dimensional.

We introduce a scalar field A and a two-form field strength F as follows.

$$ds^2 = e^{2A}(\text{AdS}_3) + g_{ab}dx^a dx^b, \quad a, b = 1, 2, \dots 7. \quad (2.1)$$

$$F^{(5)} = (1 + *)\text{Vol}_{\text{AdS}_3} \wedge F. \quad (2.2)$$

The dilatino variation vanishes trivially and we only need to consider the gravitino variation equation,

$$\nabla_M \epsilon + \frac{i}{480} \Gamma^{M_1 \dots M_5} F_{M_1 \dots M_5}^{(5)} \Gamma_M \epsilon = 0. \quad (2.3)$$

It is required to introduce a specific basis for the gamma matrices which respect the dimensional decomposition we consider here.

$$\Gamma_\mu = \sigma_1 \otimes \gamma_\mu \otimes 1, \quad \mu = 0, 1, 2. \quad (2.4)$$

$$\Gamma_a = \sigma_2 \otimes 1 \otimes \gamma_a, \quad a = 3, \dots 9. \quad (2.5)$$

In this basis the 10 dimensional chirality projection implies $\sigma_3 \epsilon = +\epsilon$. On AdS_3 the Killing spinor satisfies the following property.

$$\nabla_\mu \epsilon = \frac{a}{2} \gamma_\mu \epsilon, \quad a = \pm 1. \quad (2.6)$$

Now we can rephrase the Killing spinor equation in terms of a 7 dimensional Dirac spinor η . We obtain the following set of equations,

$$\nabla_a \eta - \frac{e^{-3A}}{4} \not{F} \gamma_a \eta = 0, \quad (2.7)$$

$$\left(\not{D} A + \frac{e^{-3A}}{2} \not{F} - i a e^{-A} \right) \eta = 0. \quad (2.8)$$

In 7 dimensions with Euclidean signature it is possible to define Majorana spinors but from the Killing equations above it is obvious we need to consider a Dirac spinor. Since η has 8 components generic Killing spinor solutions should preserve 1/8 supersymmetry. We might understand this statement as the supersymmetry of D3-branes wrapping a Kahler two-cycle in a Calabi-Yau four-fold which consists of the 2 tangential and 6 transverse directions of the D3-brane world-volume.

3. Spinor Bilinears

One can construct differential forms of various ranks defined on the 7 dimensional space as spinor bilinears.

$$C = \eta^\dagger \eta, \quad (3.1)$$

$$K_a = \eta^\dagger \gamma_a \eta, \quad (3.2)$$

$$Y_{ab} = i \eta^\dagger \gamma_{ab} \eta, \quad (3.3)$$

$$Z_{abc} = i \eta^\dagger \gamma_{abc} \eta, \quad (3.4)$$

$$W_{abcd} = \eta^\dagger \gamma_{abcd} \eta, \quad (3.5)$$

$$X_{abcde} = \eta^\dagger \gamma_{abcde} \eta, \quad (3.6)$$

$$P_{abcdef} = i \eta^\dagger \gamma_{abcdef} \eta. \quad (3.7)$$

One can also consider complex conjugate spinors to construct differential forms which are complex valued. Because of the antisymmetry of gamma matrices only 3- and 4-forms are non-vanishing¹, we define

$$\Omega_{abc} = \eta^T \gamma_{abc} \eta, \quad (3.8)$$

$$\Psi_{abcd} = \eta^T \gamma_{abcd} \eta. \quad (3.9)$$

Now one can make use of the 7 dimensional Killing equations to derive algebraic and differential relations between the spinor bilinears. For instance,

$$\begin{aligned} \nabla_a (\eta^\dagger \eta) &= \nabla_a \eta^\dagger \eta + \eta^\dagger \nabla_a \eta \\ &= -e^{-3A} F_{ab} \eta^\dagger \gamma^b \eta \\ &= \partial_a A \eta^\dagger \eta, \end{aligned} \quad (3.10)$$

which implies $\eta^\dagger \eta = e^A$.

¹In our convention the 7 dimensional gamma matrices are all antisymmetric.

Proceeding in the same way one finds that K in fact defines a Killing vector, i.e. $\nabla_{(a}K_{b)} = 0$. We choose a local coordinate patch such that $K = \partial_\psi$ and write the 7 dimensional metric as follows,

$$\begin{aligned} ds^2 &= g_{ab}dx^a dx^b \quad a, b = 1, \dots, 7. \\ &= e^{2\phi}(d\psi + B)^2 + g_{ij}dx^i dx^j, \quad i, j = 1, \dots, 6. \end{aligned} \quad (3.11)$$

ϕ is a 6-dimensional scalar which is given as the norm of K through $K^2 = e^{2\phi}$ and B is a one-form in 6 dimensions.

One can in fact see that $\phi = A$. From the algebraic Killing equation Eq.(2.8) it follows $\eta^T \eta = 0$. If we introduce a pair of Majorana spinors to write

$$\eta = \frac{1}{\sqrt{2}}(\eta_1 + i\eta_2), \quad (3.12)$$

it follows that $\eta_1^T \eta_1 = \eta_2^T \eta_2$ and $\eta_1^T \eta_2 = 0$. We also have $\eta^\dagger \gamma^i \eta = 0$, which implies $\eta_1^T \gamma^i \eta_2 = 0$. Because of the orthogonality of gamma matrices and that they are 8 dimensional, it is obvious that $\eta, \gamma^i \eta, \gamma^\psi \eta$ span the whole spinor space. Since η_1 is orthogonal to η_2 and $\gamma^i \eta_2$, one concludes that $\eta_1 \propto \gamma_\psi \eta_2$. Since η_i are real spinors of the same magnitude, we conclude $\eta_1 = \pm i \gamma_\psi \eta_2$, which implies the Dirac spinor η is chiral on 6 dimensional space defined by x^i in Eq.(3.11)². Then $K^2 = e^{2\phi} = (\eta^\dagger \gamma^\psi \eta)(\eta^\dagger \gamma_\psi \eta) = (\eta^\dagger \eta)^2 = e^{2A}$. The chiral spinor η thus defines a $SU(3)$ structure on the 6 dimensional base space, Y defines an almost complex structure and Ω is a $(3, 0)$ -form.

Above procedure of exploiting Killing equations can be repeated for other differential forms. One first uses the differential relation to compute the exterior derivative, and the result is simplified by making use of the algebraic equation. We summarize the result as follows,

$$d(e^{2A}K) = -4F - 2ae^A Y, \quad (3.13)$$

$$d(e^A Y) = 0, \quad (3.14)$$

$$d(e^{4A}Z) = 4ae^{3A}W - 4e^A Y \wedge F, \quad (3.15)$$

$$d(e^{3A}W) = 0, \quad (3.16)$$

$$d(e^{2A}X) = 2ae^A P, \quad (3.17)$$

$$dP = 0. \quad (3.18)$$

Now from the chirality condition of η in 6 dimensions all the higher forms can be written as a wedge product of K, Y and only the first two equations in the above are independent. One also derives

$$d(e^{2A}\Omega) = 2aiK \wedge \Omega. \quad (3.19)$$

Now one can easily see that for the rescaled metric $\bar{g}_{ij} = e^{2A}g_{ij}$, $J = -e^A Y$ and $\omega = e^{2A}e^{2ai\psi}\Omega$ provide the canonical two-form and the $(3, 0)$ -form of the almost complex

²Here we use the notation where the hatted indices denote orthonormal frame, i.e. $\{\gamma_{\hat{a}}, \gamma_{\hat{b}}\} = 2\delta_{\hat{a}\hat{b}}$.

structure defined by Y . In particular, using Fierz identity, one can check that

$$\begin{aligned}\text{Vol}_6 &= \frac{1}{6} J \wedge J \wedge J \\ &= \frac{i}{8} \omega \wedge \bar{\omega}.\end{aligned}\tag{3.20}$$

$dJ = 0$ and $d\omega = 2aiB \wedge \omega$ together then imply that the complex structure is integrable and $2adB$ is the Ricci-form of the Kahler manifold.

There is another equation derived from Eq.(2.8) when we multiply η^T , which relates the 6 dimensional curvature scalar with A . We do not find any other independent conditions from the Killing equations Eq.(2.7,2.8), so we here summarize the equations which guarantee the supersymmetry of the configuration,

$$F = \bar{F} + K \wedge e^{2A} dA, \tag{3.21}$$

$$ae^{4A} \mathcal{R} = -8\bar{F} + 4aJ, \tag{3.22}$$

$$R = 8e^{-4A}. \tag{3.23}$$

where \bar{F} is the two-form field restricted to 6 dimensional space and $J^{ij} = \bar{g}^{ik} \bar{g}^{jl} J_{kl}$. \mathcal{R} is the Ricci form, and R is Ricci scalar. It is clear that once the Kahler space is fixed the above equations can determine A, F , thus the entire 10 dimensional solution.

It is an established fact that a supersymmetric configuration satisfies the classical field equations provided the form-field equations of motion and the Bianchi identities are satisfied [13]. The Bianchi identity $dF = 0$ is a consequence of supersymmetry as can be easily seen from Eq.(3.13) and Eq.(3.14). The form-field equation of motion $d(e^{-3A} * F) = 0$ can be checked most easily using the second line of Eq.(3.10). As we take another covariant derivative, after some algebra one can show it leads to

$$\square R - \frac{1}{2} R^2 + R_{ij} R^{ij} = 0 \tag{3.24}$$

where the norms are taken with respect to the rescaled metric \bar{g} .

Let us summarize. We have shown that, if we restrict ourselves to pure D3-brane backgrounds, any supersymmetric solution of IIB supergravity with an AdS_3 factor can be always written,

$$ds^2 = e^{2A} ds^2(AdS_3) + e^{2A} (d\psi + B)^2 + e^{-2A} ds_{\text{Kahler}}^2 \tag{3.25}$$

$$F^{(5)} = (1 + *) \text{Vol}_{AdS_3} \wedge \left(\frac{a}{2} J - \frac{1}{4} d(e^{4A} (d\psi + B)) \right) \tag{3.26}$$

In principle, one can construct new AdS_3 solutions starting with a 6 dimensional Kahler space satisfying Eq.(3.24), then using Eq.(3.23) and $dB = 2\mathcal{R}$. Eq.(3.24) can be rewritten as a 4th-order partial differential equation for the Kahler potential. Instead of trying to solve Eq.(3.24) directly, in the remainder of this article we illustrate how well-known solutions can be rephrased in terms of our result.

4. Examples

We now construct explicit solutions from the equations presented in the last section and in particular show how the well-known AdS solutions can be rephrased in our general framework.

As the simplest case we choose the 6 dimensional Kahler basis to be Einstein or products of Kahler-Einstein spaces, i.e. $A = \text{const.}$ Eq.(3.24) then becomes a simple algebraic relation involving the dimensionalities of Kahler-Einstein manifolds,

$$R^2 = 2R_{ij}R^{ij}. \quad (4.1)$$

It is easy to see that the only possibility is $S^2 \times T^4$ if we exclude the use of hyperbolic spaces. This case corresponds to the most well-known example of IIB supergravity with a AdS_3 factor, i.e. $\text{AdS}_3 \times S_3 \times T^4$ which has $1/2$ unbroken supersymmetry. When we set $A = 0$ the radius of S^2 is $1/2$, and if we introduce θ, ϕ as the coordinates on S^2 with the standard metric, the 7 dimensional metric is written as

$$ds^2 = \frac{1}{4} \left((d\psi + \cos \theta d\phi)^2 + d\theta^2 + \sin^2 \theta d\phi^2 \right) + (T^4\text{-part}). \quad (4.2)$$

The 10 dimensional solution is interpreted as the near-horizon limit of two intersecting D3-branes on a string.

As the second example we consider the type of solutions $\text{AdS}_5 \times SE_5$ where SE_5 is a 5 dimensional Sasaki-Einstein space. The Ramon-Ramond five-form is given as the sum of AdS_5 and SE_5 volume form. These solutions correspond to the near horizon limit of D3-branes put on a singular point of Calabi-Yau space. It is well known that a Sasaki-Einstein space can be always written as a Hopf-fibration over a Kahler-Einstein space, i.e.

$$ds^2 = (d\alpha + \sigma/3)^2 + ds_{KE}^2, \quad (4.3)$$

with $d\sigma/6$ the Kahler form of the Kahler-Einstein base. The constant norm Killing vector ∂_α is called Reeb vector.

The simplest examples of Kahler-Einstein space are given from complex projective spaces CP^n . In 4 dimensions we have two obvious choices, CP^2 and $\text{CP}^1 \times \text{CP}^1$. The former gives rise to S^5 , the latter $T^{1,1}$ respectively. The metric cone of $T^{1,1}$ is the conifold and the dual gauge theory living on the D-branes put on conifold singularity is understood in detail [14].

Until recently $T^{1,1}$ has been the only 5 dimensional Sasaki-Einstein manifold whose metric is known explicitly. A couple of new, infinite class of Sasaki-Einstein manifolds were discovered recently [10, 15]. They all turned out to be toric, and the dual conformal field theories given as quiver gauge theories have been identified.

It is possible to reconstruct these solutions from our equations. It is just a matter of choosing the right 6 dimensional Kahler space. In order to determine the right Kahler space we try to rewrite $\text{AdS}_5 \times SE_5$ solutions.

$$ds^2 = ds^2(\text{AdS}_5) + ds^2(SE_5)$$

$$\begin{aligned}
&= \cosh^2 \rho ds^2(\text{AdS}_3) + d\rho^2 + \sinh^2 \rho d\phi^2 + \left(d\alpha + \frac{\sigma}{3}\right)^2 + ds^2(\text{KE}_4) \\
&= \cosh^2 \rho ds^2(\text{AdS}_3) + \cosh^2 \rho \left(d\phi + \frac{d\tilde{\alpha} + \sigma/3}{\cosh^2 \rho}\right)^2 \\
&\quad + \frac{1}{\cosh^2 \rho} \left(\cosh^2 \rho (d\rho^2 + ds^2(\text{KE}_4)) + \sinh^2 \rho (d\tilde{\alpha} + \frac{\sigma}{3})^2\right), \tag{4.4}
\end{aligned}$$

where we set $\tilde{\alpha} = \alpha - \phi$. The Kahler form of the 6 dimensional base space is written as follows,

$$J = \cosh \rho \sinh \rho d\rho \wedge (d\tilde{\alpha} + \sigma/3) + \sinh^2 \rho J_{KE}. \tag{4.5}$$

J_{KE} is the Kahler form of the 4 dimensional Kahler-Einstein manifold which gives rise to the Sasaki-Einstein manifold. One can check $\cosh^2 \rho$ and $2d\left(\frac{d\tilde{\alpha} + \sigma/3}{\cosh^2 \rho}\right)$ correctly give the scalar curvature and the Ricci-form of the 6 dimensional Kahler space.

The next example is the 1/2-BPS solutions of IIB supergravity obtained in [11]. Solutions with $SO(4) \times SO(4)$ symmetry, i.e. having two factors of three-sphere are studied and argued to be dual to generic 1/2-BPS operators of $N = 4$ super Yang-Mills theory. Our result can be easily translated into the case of IIB solutions with one S^3 instead of AdS_3 through double Wick rotation. The spacelike Killing vector becomes a timelike Killing vector fibred again over a 6 dimensional Kahler manifold. The solution takes the following form,

$$\begin{aligned}
ds^2 &= -h^{-2}(dt + V_i dx^i)^2 + h^2(dy^2 + dx^i dx^i) + ye^G d\Omega_3^2 + ye^{-G} d\tilde{\Omega}_3^2, \\
h^{-2} &= 2y \cosh G, \\
y\partial_y V_i &= \epsilon_{ij} \partial_j z, \quad y(\partial_i V_j - \partial_j V_i) = \epsilon_{ij} \partial_y z, \\
z &= \frac{1}{2} \tanh G, \\
F &= dB_t \wedge (dt + V) + B_t dV + d\hat{B}, \\
\tilde{F} &= d\tilde{B}_t \wedge (dt + V) + \tilde{B}_t dV + d\hat{\tilde{B}}, \\
B_t &= -\frac{1}{4} y^2 e^{2G}, \quad \tilde{B}_t = -\frac{1}{4} y^2 e^{-2G}, \\
d\hat{B} &= -\frac{1}{4} y^3 *_3 d\left(\frac{z + 1/2}{y^2}\right), \quad d\hat{\tilde{B}} = -\frac{1}{4} y^3 *_3 d\left(\frac{z - 1/2}{y^2}\right), \tag{4.6}
\end{aligned}$$

where F, \tilde{F} are defined from the dimensional reduction of Ramond-Ramond 5-form through

$$F_{(5)} = F_{\mu\nu} dx^\mu \wedge dx^\nu \wedge d\Omega_3 + \tilde{F}_{\mu\nu} dx^\mu \wedge dx^\nu \wedge d\tilde{\Omega}_3. \tag{4.7}$$

The full solution is determined by a single function $z(x^1, x^2, y)$, which satisfies the following equation.

$$\partial_i \partial_i z + y \partial_y \left(\frac{\partial_y z}{y}\right) = 0. \tag{4.8}$$

In order to give a regular solution it is argued in [11] that $z = \pm \frac{1}{2}$ on the $y = 0$ plane. It turns out that once the the shape of the filled region defined by $z = \frac{1}{2}$ on the (x^1, x^2) -plane is specified the full 10 dimensional solution is determined. Then the filled

region is interpreted as the fermi sea of the (fermionized) Yang-Mills eigenvalues. Since our result applies to any supersymmetric solutions, any solutions satisfying Eqs.(4.6),(4.8) can be written based on a 6 dimensional Kahler manifold.

We first write the metric of \tilde{S}^3 in terms of left-invariant forms

$$d\tilde{\Omega}_3^2 = \frac{1}{4}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2), \quad (4.9)$$

and introduce polar coordinates for the 2 dimensional space (x^1, x^2) as

$$dx^i dx^i = dr^2 + r^2 d\phi^2. \quad (4.10)$$

In order to see the hidden Kahler structure it turns out useful to mix the two Killing vectors ∂_t and σ_3 . When we rewrite $\sigma_3 \rightarrow \sigma_3 + 2dt$, the metric can be written as

$$ds^2 = ye^G d\Omega_3^2 - ye^G (dt + \frac{V}{h^2 ye^G} - \frac{\sigma_3}{2e^{2G}})^2 + \frac{1}{ye^G} \left[h^2 ye^G (dr^2 + r^2 d\phi^2) + \frac{y}{4h^2 e^G} (\sigma_3 - 2V)^2 + h^2 ye^G dy^2 + \frac{y^2}{4} (\sigma_1^2 + \sigma_2^2) \right] \quad (4.11)$$

The Kahler form is given as

$$J = -(z + \frac{1}{2})rdr \wedge d\phi + \frac{y^2}{4}\sigma_1 \wedge \sigma_2 + \frac{y}{2}dy \wedge (\sigma_3 - 2V). \quad (4.12)$$

One can easily check it is indeed closed, and compute the Ricci-form to show that it is given as $d(\frac{2}{2z+1}V - e^{-2G}\sigma_3)$, and Eq.(3.24) is satisfied.

5. Discussion

In this work we analyzed the D3-brane configurations which lead to AdS_3 solutions. The full 10 dimensional solution is determined once a 6 dimensional Kahler space is chosen which satisfies a certain condition for the curvature, given in Eq.(3.24). With appropriate Kahler manifolds we can reconstruct known IIB solutions which contains an AdS_3 factor. Let us emphasize that, since the higher dimensional AdS spaces can be written as a bundle over a lower dimensional AdS, we can describe all higher dimensional AdS spaces from 5-forms in terms of our result.

It would be very interesting to construct new AdS_3 solutions using our result and analyze their gauge theory duals. One can use hyperbolic Kahler manifolds as part of the 6 dimensional Kahler base manifold to find more solutions to Eq.(4.1). This is far from surprising, and in fact very reminiscent of the general results of wrapped brane solutions reported in [16], where numerous AdS solutions have been obtained through wrapping branes on hyperbolic supercycles.

It is straightforward but very interesting to consider the Wick rotated case of our solutions: instead of assuming an AdS_3 factor one considers S_3 . Supersymmetry and the $SO(4)$ isometry imply that in the dual picture these solutions can describe pure scalar operators of $N = 4, D = 4$ super Yang-Mills which are in general 1/8-BPS. They can also describe 1/2-BPS operators of certain $N = 1, D = 4$ superconformal field theories.

From the analysis in Ref.[11] we know that part of the string theory spacetime can be mapped to the phase space of the Yang-Mills eigenvalue dynamics. According to our result supersymmetry requires that the phase space of eigenvalue dynamics for generic BPS states should have not only symplectic, but Kahler structure. It will be very exciting if one can find a prescription to extract the information on the dual gauge theory and the fluctuation thereof, from the 6 dimensional Kahler geometry. The geometric constraint Eq.(3.24) would tell us how the eigenvalue distribution is transcribed into general relativity. The relevant matrix model description of the eigenvalue dynamics for $N = 4, D = 4$ Yang-Mills theory has been discussed in [17]. We hope to be able to address this issue further in the near future.

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